Identification, shaft, turning, grinding, systems analysis, hierarchy

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SYSTEMIC ANALYSIS OF MODELS OF DYNAMIC SYSTEMS OF SHAFT MACHINING IN ELASTIC-DEFORMABLE CONDITION

Abstract

The paper presents methodology of developing models of dynamic systems of machining shafts in the elastic-deformable condition. The specifics of dynamic system (DS) identification concerning straight turning and straight and plunge grinding of low-rigidity shafts is presented. The specifics of the process of low-rigidity elements machining is taken into account through the introduction of suitable equations of constraint reflecting additional elastic strain in one of the equations describing the control force effect. Systemic analysis of the developed models is performed, and their hierarchical structure is given.

1. INTRODUCTION

Continued efforts aimed at obtaining high-quality machining on machine tools under conditions of various interferences affecting the technological system (*TS*) have led to the application of adaptive control (*AC*) systems in the machine-building industry [1, 2]. The problem of improvement of such systems is particularly relevant under *ESP* conditions, in the realization of so-called "no-man" technology. Development of a mathematical model (*MM*) of control object (*CO*) in the dynamics, adequate to the original object, is a prerequisite for substantiated approach to the solution of the stability analysis problem of automatic control systems (*ACS*) or *AC* and synthesis of correcting elements, in accordance with required quality indices of transition process control. Whereas, in similar systems, indexes of quality of control of the input variable – elastic deformation in the dynamic *TS* - characterize directly the errors of the machined parts shape, determined by the effect of rapidly changing interferences of the type of change in material allowance for machining or variability of the physicochemical properties of the machined material.

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The dynamic system (*DS*) of the process of machining is a technological system – MHWT (Machine tool, Holder, Workpiece, Tool), i.e. a machine tool together with the realized technological process (*TP*) of machining (turning, grinding, drilling, milling) [3, 4].

Systemic analysis, as the basis for analysis and synthesis of *ACS* and *AC,* is based on the assumption that the specifics of the assumed objects and processes results not only from the properties of the component elements, but is determined by the character of their mutual relationships that have a decisive effect on the structure of the *ACS* or *AC*.

In the identification of *DS* the systemic approach includes the following fundamental stages [17]:

- analysis of input data for the identification;
- formulation of control strategy oriented at a specific subsystem of basic machine tools, in accordance with input data in designing *ACS*, *AC*;
- exclusion of invariant, relative to their spectrum, input effects of subsystems and components within the limits of technical capability of *ACS*, *AC* and the machine tools;
- analysis of possible structures of *MM* of control system with respect to their function, types of components and connections between them, number of hierarchy levels, principles of connection, and permanence of the connections.

With a lack of sufficiently complete and detailed information on the object of control, calculated characteristics may significantly differ from the true ones. The parameters (settings) of regulators adopted in designing do not guarantee the required quality of control, or even stability of the system. Apart from this, the analysed systems are characterized by extensive variability of parameters of the *CO*. In technological systems comprising a semi-finished product of low rigidity the parameters may change notably within the machining cycle of a single part. Those determinations indicate the complexity of the problem of ensuring stability of the *ACS* and the necessity of taking special care in the approach to the problem of defining its structure and synthesis of the corrective devices.

2. IDENTIFICATION OF DYNAMIC SYSTEMS OF SHAFT TURNING

In the case when there is complete information on the object of control it is possible to design a model using the analytical method. Such a procedure, leading to the identification of the structure and parameters of a model, is referred to as the analytical identification. For complex systems, development of *MM* with the analytical method frequently requires additional experimental tests aimed at the verification of theoretical results and at determination of some of the model parameters.

The presented schematic of the structure of *MM* shows that the basic scope of work in the design of *MM* is based on in-depth theoretical analysis of connections between the variable parameters and on revealing the relationships describing the processes taking place within the object.

The possibility of linearization of the particular *DS* components motion equations follows also from the commonly accepted view that assurance of high requirements with respect to precision of adjustment is reduced to realization of adjustment systems operating at “small” deviations of variables. Therefore, the dynamic system of the drilling process can be considered as multi-dimensional *CO* with subsystems in the form of the technological process and an elastic system. The structure of the *CO* includes circuits of feedbacks from the elastic system caused by force effects that appear in the course of realization of the technological process.

References [4, 5, 6] present a system of equations and a generalized structural schematic of *MM* of the dynamic system of shaft turning. The developed system of equations and the structural schematic of *MM* take into account the geometry of the machined layer and of the machining force in turning, elastic properties of the *TS*, process of forming cross-section of the machined layer (*ML*). The process of forming cross-section of the ML takes into account the phenomenon of machining "following the feed ridge" which consists in that the components of the machined layer of the material at the current moment are defined by the temporary position of the cutting edge and by its coordinates at the moment of the preceding revolution of the semi-finished product, i.e. at a time-lag of a single revolution. At the same time the effect of elastic deformation for coordinate Z on the depth of turning is taken into account.

The process of forming of the cross-section of *ML* is under strong effect of the phenomenon of machining "following the feed ridge" and by elastic deformations in the *DS*. The process of forming of ML cross-section can be described with a system of integral-differential equations with delayed argument. Variables characterizing the *ML* cross-section depend on the input variables and on the elastic deformation in the *DS*. In the vector of the technological variables, formed by the dynamic system, two components can be distinguished – one defined by the vector of input effects and the other by the vector of elastic deformations.

Elements of the vector of input values are the control values in the form of the straight feed rate, rotational speed of the machined part, and also interference in the form of hardness changes of the machined material and in the machining allowance relative to the length and diameter of the machined part.

The vector of elastic deformations is determined by the vectors of machining forces and of control values entering the system of vibrational stability assurance. Dynamic properties of the equivalent elastic system can be approximated with quadratic equations [7]. The choice of the vector of technological variables is significantly affected by the phenomenon of machining "following the feed ridge", manifest in that the momentary values of the components of the said vector are determined by the values of elements of the input vector and of the vector of elastic deformations not only at the current moment but also at the time of the preceding revolution of the machined part. Due to this the dynamic system is described with a system of integral-differential equations with variable delayed argument.

As a result of analysis of the processes occurring in the dynamic system of machining a system of equations and functions of transition were obtained, as well as the generalized structure of the control object.

2.1. Identification of DS of turning of low-rigidity shafts

To improve the precision of machining of shafts with low rigidity, technological methods were developed for the control of machining precision, based on change in the elastic-deformable status [9, 10]. As control effects, in accordance with the developed classification [9], particular force control effects are employed, or their combinations – axial and eccentric tension, control by means of additional force effects aimed at compensation of force factors from the machining process, bending moments at supports, control of force-induced bending-torsional strain.

MM of various technological systems of machining with control of the elastic-deformable status for stabilised parameters, presented in the form of deflection functions, were obtained with the assumption that a banding force acting on the machined part is an external variable that is independent of the elastic deformations in the *DS*. This approach is based on not including the closing of the elastic system through the process of machining and does not introduce new errors into results of analyses of static characteristics of the *CO*. Analysis of the structure of a suitable *MM* of a control object for transition parameters is not possible without taking into consideration the specifics of processes within the machining zone and the closing of the *DS* through the process of machining.

MM of the considered control object – *DS* with control of the elastic-deformable status of parts with low rigidity was constructed on the basis of general principles of creating *MM* of *DS* [4, 5] of machining, with the specifics of the process of machining of parts with low rigidity being accounted for by the introduction of suitable equations of constraints [11, 12], reflecting mutual relationships between additional elastic deformations Δg_{ξ}, into one of the equations representing the force control effects of the system of equations.

Equivalent elastic deformations of the *TS* in the machining of parts with low rigidity can be represented in the form of two components:

$$g_{\zeta} = g_{\zeta obr.} + g_{\zeta cz.}, \tag{1}$$

where: $g_{\zeta obr.}$ and $g_{\zeta cz.}$ - elastic deformations of the machine tool, fixture, tool and part for each coordinate, respectively; $\zeta \in \{x, y, z\}$. The first component in this expression for the *TS* under consideration is, in principle, lower by one order of magnitude and can be neglected.

Elastic deformations of the *TS* in the radial direction g_y in accordance with the deflection equations [9], at set parameters without the inclusion of closed status of the *CO*, may be considered as a deterministic non-linear function of the part parameters L, d, EI; components of the machining force F_c, F_p, F_f; coordinates x of machining force application on the length of the semi-finished product and various regulatory effects in the form of: tensile force F_{x1}; eccentric tensile force creating two regulatory effects F_{x1} and moment $M = F_{x1} \cdot e$, where e - eccentric of the tensile forces; one or more additional forces $F_{dod.i}$; bending moments M_i; torsional moment M_{skr} or their combinations:

$$g = f(L, d, EI, F_c, F_p, F_f, F_{x1}, e, F_{dod.i}, M_i, M_{skr}, x) \,. \tag{2}$$

Assuming that the true feed rate and the rate of change of coordinate x are relatively small, in the analysis of transition processes the change in coordinate x in the function of time can be left out. Therefore, relation (2) in the operator form can be written as:

$$\begin{aligned} g_y(s) &= K_{xy} \cdot F_f(s) + K_{yy} \cdot F_p(s) + K_{zy} \cdot F_c(s) + K_{F_{x1}} \cdot F_{x1}(s) + \\ &+ K_e \cdot e(s) + K_{F_{dod.i}} \cdot F_{dod.i}(s) + K_{M_i} \cdot M_i(s) + K_{M_{skr}} \cdot M_{skr}(s) \end{aligned} \,, \tag{3}$$

where: dual indexes at coefficients K mean that coefficients K_{xy}, K_{zy} indicate the effect of increase in the values of components F_f, F_c on increase in the level of elastic strain on coordinate y; $K_e = K_e^{'} \cdot F_{x1_0}$. The gain coefficients of linear equations are defined as fragmentary derivatives of the strain function along the respective coordinate. For example, for the *TS* of machining with the effect of axial tensile force F_{x1}, causing the elastic-deformable state, from the system of elastic deformations we obtain [9]:

$$K_{yy} = \left(\frac{\partial g_y}{\partial F_p}\right)_0 = \frac{L^3 \cdot [1-\cos(2\pi x_0 / L)]^2}{2\pi^2 \cdot (4\pi^2 \cdot EI + F_{x1_0} \cdot L^2)}, \tag{4}$$

$$K_{F_{x1}} = \left(\frac{\partial g_y}{\partial F_{x1}}\right)_0 = -\frac{F_p \cdot L^5 [1-\cos(2\pi x_0 / L)]^2}{2\pi^2 \cdot (4\pi^2 \cdot EI + F_{x1} \cdot L^2)} = -\frac{g_{y_0} \cdot L^2}{4\pi^2 \cdot EI + F_{x1} \cdot L^2}, \tag{5}$$

where: F_{x1_0}, g_{y_0} - values of tensile force and elastic strain of the part along coordinate y at the point of linearization (values of variables relative to which increases of variables are given). In the special case under consideration the remaining coefficients in relation (3) are equal to zero. Coefficients of gain, corresponding to different *DS* at various methods of loading (i.e. with axial-radial bending and various methods of fixing) in machining of elastic-deformable parts, obtained in an analogous manner, are presented in Table 2 – column 2, x_0 - coordinate of cutting edge position on machining length at the point of linearization [13]. The additional elastic strains g_x, g_z with respect to coordinates x and z, as a result of the action of the control force effects under consideration, basically do not have any significant effect on the dynamic properties of the *CO* and can be treated as negligible.

In accordance with the result of studies in ref. [14], the components of machining force without inclusion of the contact strain at the surface of application are written as:

$$F_c = Q_{pw} \cdot a \cdot b, \quad F_p = Q_{pw} \cdot a \cdot b \cdot K'_y, \quad F_f = Q_{pw} \cdot a \cdot b \cdot K'_x,$$

where: Q_{pw} - relative work of formation of shaving, K'_y, K'_x - constant coefficients for given conditions of machining.

Hence

$$m_z = \left(\frac{\partial F_c}{\partial a}\right)_0 = Q_{pw_0} \cdot b_0 \cdot K_z, \; m_y = \left(\frac{\partial F_p}{\partial a}\right)_0 = Q_{pw_0} \cdot b_0 \cdot K_y, \; m_x = \left(\frac{\partial F_f}{\partial a}\right)_0 = Q_{pw_0} \cdot b_0 \cdot K_x,$$

$$n_z = \left(\frac{\partial F_c}{\partial b}\right)_0 = Q_{pw_0} \cdot a_0 \cdot K_z, \; n_y = \left(\frac{\partial F_p}{\partial b}\right)_0 = Q_{pw_0} \cdot a_0 \cdot K_y, \; n_x = \left(\frac{\partial F_f}{\partial b}\right)_0 = Q_{pw_0} \cdot a_0 \cdot K_x$$

and

$$n_y m_x = Q_{pw_0} a_0 K_y Q_{pw_0} b_0 K_x, \quad m_z n_x = Q_{pw_0} b_0 K_z Q_{pw_0} a_0 K_x,$$
$$m_y n_x = Q_{pw_0} b_0 K_y Q_{pw_0} a_0 K_x, \quad n_z m_x = Q_{pw_0} a_0 K_z Q_{pw_0} b_0 K_x,$$
$$n_y m_x = m_y n_x, \quad m_z n_x = n_z m_x.$$

The relations given above permit simple transformations of coefficients A and B included in corresponding operator transmittances (*OT*) of the *CO* with relation to various control and interfering effects.

In referenced works [15, 16] the authors analysed the possibility of replacing the obtained relations of *TO* with approximated ones, application of which significantly simplifies calculation of characteristics of *DS MM*. The analysis was made according to the criterion of recreation of true characteristics of *MM* with approximated relations in the time and frequency planes; it was demonstrated that the form of approximating relations should be chosen taking into account the numerical value of coefficient B. It was also determined that the value of

$B = 0{,}1$ is the “limit” at which the switch from one form of approximating relation to another is justified. The value of coefficient B is defined as the ratio of rigidity of equivalent elastic system to gain coefficients of the process of machining and can be adopted as an index of relative rigidity of *DS*. Broad ranges of variability of machining parameters on machine tools, e.g. of change in the hardness of the machined material, machining allowance, cutting edge geometry, determine broad ranges of variability of coefficients $m_x, m_y, K_{\kappa_r}, K_x, K_{yy}$ and B, respectively.

Calculations show that in machining of low-rigidity shafts and in roughing and profiling of parts with normal rigidity the values of coefficient B are notably greater than the limit value of $B = 0{,}1$; in this case also the approximating relations for *TO* according to (11), (14), (15) should be built by splitting the exponential function $e^{-s\tau}$ into a Padé series which, keeping the first two components, may be written as:

$$e^{-s\tau} = \left(1 - \frac{1}{2}s \cdot \tau + \frac{1}{12}s^2 \cdot \tau^2\right) \Big/ \left(1 + \frac{1}{2}s \cdot \tau + \frac{1}{12}s^2 \cdot \tau^2\right). \tag{6}$$

In the case of control of the elastic-deformable state of parts with low rigidity through the application of tensile force F_{x1} the structure of *CO* has been developed in [13].

On the basis of the schematic given in [13], after transformation, the relation for *TO* of the dynamic system when increase in elastic deformations g_y in the radial direction is adopted as the initial variable is reduced to the form of:

$$G_{F_{x1}}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{1 + A' \cdot (1 - e^{-s\tau})}{1 + B' \cdot (1 - e^{-s\tau})}, \tag{7}$$

where:

$$K_0 = K_{F_{x1}} \cdot \frac{1}{1 + K_{yy} \cdot n_y + K_{xy} \cdot n_x + K_{bz} \cdot K_z \cdot n_z}, \tag{8}$$

$$A' = m_x \cdot K_x + K_{\kappa_r} \cdot m_y \cdot K_y, \tag{9}$$

$$B' = \frac{m_x \cdot K_x + K_{\kappa_r} \cdot m_y \cdot K_{yy} \left[2 + K_{yy} \cdot n_y + K_{bz} \cdot n_z + K_{xy} \cdot m_x /(K_{yy} \cdot m_y) + K_{bz} \cdot K_z \cdot m_z /(K_{yy} \cdot m_y)\right]}{1 + K_{yy} \cdot n_y + K_{xy} \cdot n_x + K_{bz} \cdot K_z \cdot n_z}. \tag{10}$$

For known values of coefficients included in relations (7) – (10), the relations can be notably simplified. Calculations show that in machining of parts with low rigidity with application of force effects components containing K_{bz} and K_{xy} can be basically left out. In such a situation, the relation for B' gets considerably simplified, and the expression for coefficients K_0 is notably reduced. Denominator of *TO* of operator transmittance for *DS* determined from the relation in control of straight feed [4, 6] is reduced, as shown above, to the form of denominator of aperiodic component of the first or second order. To transform the numerator of *TO* to a typical form one can also employ splitting the function $e^{-s\tau}$ into a Padé series, and then the analysed *TO* will assume the form of:

$$G_{F_{x1}}(s) = K_0 \cdot \frac{T_3^2 \cdot s^2 + T_3' \cdot s + 1}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)}, \tag{11}$$

The time constants T_1 and T_2 are determined from the relation:

$$T_{1,2} = 0{,}5\tau \cdot \left[0{,}5 + B \pm \sqrt{(0{,}5 + B)^2 - 1/3}\right] \tag{12}$$

by substituting in it B' to replace B, and the time constants in the numerator are then equal to:

$$T_3 = 0{,}289\tau; T_3' = (0{,}5 + A') \cdot \tau. \tag{13}$$

2.2. Simplification of MM of dynamic system of shaft turning in the elastic-deformable state

Further transformations of the numerator of *TO* (7) should be made with the inclusion of time constants T_3 and T_3' which depend on A'. If $A' < 0{,}077$, then the *TO* of *UD* can be written in the following typical form:

$$G_{F_{x1}}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{T_3^2 \cdot s^2 + 2\varepsilon \cdot T_3 \cdot s + 1}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)}, \tag{14}$$

where: ε - coefficient of attenuation

$$\varepsilon = \frac{0{,}5 + A'}{0{,}577}. \tag{15}$$

In the case when $A' \geq 0{,}078$, the approximating relation for the analysed *TO* assumes the form of:

$$G_{F_{x1}} = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{(T_4 \cdot s + 1) \cdot (T_5 \cdot s + 1)}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)}, \tag{16}$$

where: $T_{4,5} = 0{,}5\tau \cdot \left[0{,}5 + A' \pm \sqrt{(0{,}5 + A')^2 - 1/3} \right]$.

In an analogous way, on the basis of the generalized structural schematic and system of equations [13] models of *DS* were obtained for other control effects. The approximating relations of dynamic system *TO* for various control effects differ from those presented here only in the value of the gain coefficient K_0 of the *CO*. Instead of coefficient $K_{F_{x1}}$ in relation (8) for K_0, in such a case coefficients of gain for the respective effects $K_e, K_{F_{dod.i}}, K_{M_i}, K_{M_{skr.}}$ are inserted. The values of those coefficients can be calculated according to the relations given in [13].

In many cases, with accuracy sufficient for practical engineering calculations, approximating relations for *TO* (7) should be built with the use of the first component of the splitting of function $e^{-s\tau}$ into a Padé series:

$$e^{-s\tau} = (1 - \frac{1}{2} s \cdot \tau)/(1 + \frac{1}{2} s \cdot \tau) \quad . \tag{17}$$

Table 1 presents operator transmittances, coefficients of gain and time constants for the generalized and the detailed *MM* of dynamic system for the turning of low-rigidity shafts in the elastic-deformable state.

3. IDENTIFICATION OF DS OF STRAIGHT GRINDING OF SHAFTS WITH LOW RIGIDITY

Taking into account the research presented in reference [5], the scheme of the process of formation of shaving in straight turning is characterized by elastic bonds in the radial and axial directions, characteristic of machining processes (processes of forming the cross-section of machined layer), and by interference effects. As the input effects of the considered *DS* the following were adopted: tensile force F_{x1} in axial tension; tensile force F_{x1} and eccentric e in non-axial tension and compression; bending moments M_1 and M_2 applied to the faces of the parts. The output variables of the *DS* are the particular component forces F_p, F_f, F_c of the machining force and the corresponding elastic deformations of the *DS*: g_y, g_x, g_z.

The generalized and fragmentary model of the *DS* of grinding were built with the following initial assumptions:

- the technological process is considered to be continuous during the machining of a single part; machining is realized at constant rate v_c =const;
- the grinding wheel works basically in the self-sharpening mode maintaining practically constant level of machining capabilities, and its linear wear during the machining cycle of a single part is negligible and can be assumed to be equal to zero;
- the initial conditions are determined at the moment of grinding wheel contact with the machined surface and appearance of strain in the technological system;
- the coefficients of gain of the elastic system K_y, K_x and linear elastic deformations of *TS* along coordinates Y and X in time and on the length of machined part are taken into consideration;

– the process of machining itself is non-inertial, and the effect of "feed ridges" is taken into account.

The machining force and its particular components F_p, F_f, F_c, at assumed hardness of material of the machined part, are determined by the current parameters of reduced cross-section of machined layer $a(t)$ and $b(t)$. The machined layer thickness $a(t)$ is taken to mean the reduced thickness of machined shaving of metal which in fact is determined by the parameters of the uncountable set of micro-shavings removed by the elementary grains of the grinding wheel at the current moment [5, 6]. The cross-section of the machined layer is characterized by the current values of the reduced thickness of machined layer $a(t)$ and certain averaged values of machining depth $b(t)$ on a section of length $a(t)$, taking into account the deformations of the dynamic system along coordinate Y, determined in accordance with suitable relations [4, 5]. Characteristic for the process of grinding, as for the process of turning, is the effect of feed ridges, so-called machining "following the feed ridge". It consists in that the parameters of the machined layer are determined by the positioning of the grinding wheel cutting edge at the current moment t as well as at moment $t-\tau$ – of the preceding revolution of the semi-finished product (in the case of constant rpm of the spindle the lag time $\tau = 1/n_{wr}$).

In accordance with the system of equations [5] a generalized structural schematic of DS was built for cylindrical oscillation grinding of low-rigidity shafts (Fig.1). Analysis of the schematic shows that the reduced thickness of machines layer is determined by two components - $a_0(s)$, determined by the travel rate of the saddle, and $a_x(s)$, resulting from elastic deformations of the system along coordinate X.

Under stabilised conditions the component $a_x(s)$ does not occur, as the coefficient of gain of the component with transmittance is equal to zero $(1-e^{-s\tau})$. This is in complete agreement with the physical picture of phenomena in oscillation grinding of low-rigidity shafts, as under stabilised conditions the machined layer thickness is equal to the adopted value of feed per one revolution $\tau \cdot v_f$.

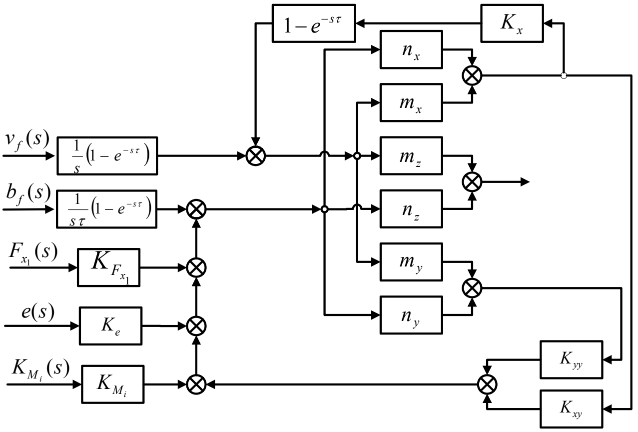

Fig. 1. Generalized structural schematic of technological system in oscillation grinding of elastic-deformable shafts with low rigidity

Table 1. Operator transmittances, coefficients of gain and time constants of generalized and simplified MM of dynamic system of turning of shafts with low rigidity in elastic-deformable condition

No	$K_{\kappa_r} \neq 0, \quad \kappa_r \neq 90^\circ$			$K_{\kappa_r} = 0, \quad \kappa_r = 90^\circ$		
	Dynamical System Operator Transmittance	**Coefficient of Gain**	**Time Constants**	**Dynamical System Operator Transmittance**	**Coefficient of Gain**	**Time Constants**
1	**2**	**3**	**4**	**5**	**6**	**7**
1	**Using first two elements of Padé Approximation for** $e^{-s\tau}$**:** $G_{T1}(s) = K_0 \dfrac{T_3^2 s^2 + T_3' s + 1}{(T_1 s+1)(T_2 s+1)}$	$K_0 = \dfrac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1 = m_x K_x + m_y K_y K_{\kappa_r}$ $B_1 = \lfloor m_x K_x + K_{\kappa_r}(m_y K_y + m_x K_{xy} +$ $+ K_{xy}n_x K_y m_y) + K_{bz}K_z K_{\kappa_r}(m_z +$ $+ n_z m_y K_y + K_{yy}m_y K_{\kappa_r}(1+n_y K_y)\ \rfloor /$ $/(1+K_{xy}n_x + K_{bz}K_z n_z + K_{yy}n_y)$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_1 \pm$ $\pm\sqrt{(0{,}5+B_1)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $T_3' = (0{,}5 + A_1)\tau$	**Using first two elements of Padé Approximation for** $e^{-s\tau}$**:** $G'_{T1}(s) = K_0 \dfrac{T_3^2 s^2 + T_3' s + 1}{(T_1 s+1)(T_2 s+1)}$	$K_0 = \dfrac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1' = m_x K_x$ $B_1' = \dfrac{m_x K_x}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_1' \pm\right.$ $\left.\pm\sqrt{(0{,}5+B_1')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $T_3' = (0{,}5 + A_1')\tau$
1	$m_x K_x << 1$	$K_0 = \dfrac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$ $B_2 = \{K_{\kappa_r}(m_y K_y + K_{xy}n_x m_y K_y) +$ $+ K_{xy}m_x(K_{\kappa_r} - n_x K_x) + K_{bz}n_z K_z \times$ $\times \lfloor K_{\kappa_r}n_z m_y K_y + m_z(K_{\kappa_r} - n_x K_x)\ \rfloor +$ $\times \lfloor K_{\kappa_r}n_z m_y K_y + m_z(K_{\kappa_r} - n_x K_x)\ \rfloor +$ $/(1+K_{xy}n_x + K_{bz}K_z n_z + K_{yy}n_y)$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_2 \pm$ $\pm\sqrt{(0{,}5+B_2)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $T_3' = (0{,}5 + A_2)\tau$	$G'_{T1}(s) = K_0 \dfrac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s+1)(T_2 s+1)}$	$K_0 = \dfrac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_2' = 0$ $B_2' = n_x K_x(m_z K_{bz}K_z + m_x K_{xy} +$ $+ m_y K_{yy})/$ $/(1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y)$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_2' \pm\right.$ $\left.\pm\sqrt{(0{,}5+B_2')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $T_3' = 0{,}5\tau$ $\varepsilon = \dfrac{T_3'}{2T_3} = 0{,}866$
1	$m_x K_x << 1, \quad K_{bz}n_z K_z << 1$	$K_0 = \dfrac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$ $B_3 = \{K_{\kappa_r}(m_y K_y + K_{xy}n_x m_y K_y) +$ $+ K_{xy}m_x(K_{\kappa_r} - n_x K_x) +$ $+ K_{yy}m_y(K_{\kappa_r} - n_x K_x)\}/$ $/(1+K_{xy}n_x + K_{yy}n_y)$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_3 \pm$ $\pm\sqrt{(0{,}5+B_3)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $T_3' = (0{,}5 + A_2)\tau$		$K_0 = \dfrac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$ $A_2' = 0$ $B_3' = n_x K_x(m_x K_{xy} + m_y K_{yy})/$ $/(1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y)$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3' \pm\right.$ $\left.\pm\sqrt{(0{,}5+B_3')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $T_3' = 0{,}5\tau$ $\varepsilon = 0{,}866$

Table 1. Operator transmittances, coefficients of gain and time constants of generalized and simplified MM of dynamic system of turning of shafts with low rigidity in elastic-deformable condition (continued)

1	2	3	4	5	6	7
1	$K_{xy} << 1$	$K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$ $B_4 = [K_{yy}m_y K_{\kappa_r}(n_y K_y + 1) +$ $+ K_{yy}m_y(K_{\kappa_r} - n_x K_x)/(1+K_{yy}n_y)$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_4 \pm$ $\pm \sqrt{(0{,}5+B_4)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $T_3' = (0{,}5 + A_2)\tau$		$K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$ $A_2' = 0$ $B_4' = n_x K_x K_{yy} m_y/(1+K_{yy}n_y)$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3' \pm\right.$ $\left.\pm \sqrt{(0{,}5+B_3')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $T_3' = 0{,}5\tau$ $\varepsilon = 0{,}866$
2	$A_1 << 0{,}077$ $G'_{T1}(s) = K_0\frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s+1)(T_2 s+1)}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1 = m_x K_x + m_y K_y K_{\kappa_r}$, $B_1 = B_1$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_1 \pm$ $\pm \sqrt{(0{,}5+B_1)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $\varepsilon = \frac{0{,}5 + A_1}{0{,}577}$	$G'_{T1}(s) = K_0\frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s+1)(T_2 s+1)}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1' = m_x K_x$ $B_1' = \frac{m_x K_x}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_1' \pm\right.$ $\left.\pm \sqrt{(0{,}5+B_1')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $\varepsilon = \frac{0{,}5 + A_1}{0{,}577}$
2	$m_x K_x << 1$, $A_2 << 0{,}077$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_2 = B_2$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_2 \pm$ $\pm \sqrt{(0{,}5+B_2)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $\varepsilon = \frac{0{,}5 + A_2}{0{,}577}$	$G'_{T1}(s) = K_0\frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s+1)(T_2 s+1)}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_2' = 0$, $B_2' = B_2'$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_2' \pm\right.$ $\left.\pm \sqrt{(0{,}5+B_2')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $\varepsilon = 0{,}866$
2	$m_x K_x << 1$, $K_{bz}n_z K_z << 1$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_3 = B_3$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_3 \pm$ $\pm \sqrt{(0{,}5+B_3)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $\varepsilon = \frac{0{,}5 + A_2}{0{,}577}$		$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$ $A_2' = 0$, $B_3' = B_3'$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3' \pm\right.$ $\left.\pm \sqrt{(0{,}5+B_3')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $\varepsilon = 0{,}866$
2	$K_{xy} << 1$	$K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_4 = B_4$	$T_{1,2} = 0{,}5\tau[0{,}5 + B_4 \pm$ $\pm \sqrt{(0{,}5+B_4)^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $\varepsilon = \frac{0{,}5 + A_2}{0{,}577}$		$K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$ $A_2' = 0$, $B_4' = B_4'$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_4' \pm\right.$ $\left.\pm \sqrt{(0{,}5+B_4')^2 - 1/3}\ \right]$ $T_3 = 0{,}289\tau$ $\varepsilon = 0{,}866$

Table 1. Operator transmittances, coefficients of gain and time constants of generalized and simplified MM of dynamic system of turning of shafts with low rigidity in elastic-deformable condition (continued)

1	2	3	4	5	6	7
3	$A_1 \geq 0{,}078$ $G_{T2}(s) = \frac{(T_4 s+1)(T_5 s+1)}{T_1 s+1)(T_2 s+1)}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1 = m_x K_x + m_y K_y K_{\kappa_r}$, $B_1 = B_1$	$T_{4,5} = 0{,}5\tau[0{,}5 + A_1 \pm$ $\pm\sqrt{(0{,}5+A_1)^2 - 1/3}\ ]$ $T_{1,2} = 0{,}5\tau[0{,}5 + B_1 \pm$ $\pm\sqrt{(0{,}5+B_1)^2 - 1/3}\ ]$	$G'_{T2}(s) = \frac{(T_4 s+1)(T_5 s+1)}{T_1 s+1)(T_2 s+1)}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1' = m_x K_x$, $B_1' = B_1'$	$T_{4,5} = 0{,}5\tau[0{,}5 + A_1' \pm$ $\pm\sqrt{(0{,}5+A_1')^2 - 1/3}$ $T_{1,2} = 0{,}5\tau[0{,}5 + B_1' \pm$ $\pm\sqrt{(0{,}5+B_1')^2 - 1/3}$
3	$m_x K_x << 1$, $A_2 \geq 0{,}078$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_2 = B_2$	$T_{4,5} = 0{,}5\tau[0{,}5 + A_2 \pm$ $\pm\sqrt{(0{,}5+A_2)^2 - 1/3}\ ]$ $T_{1,2} = 0{,}5\tau[0{,}5 + B_2 \pm$ $\pm\sqrt{(0{,}5+B_2)^2 - 1/3}\ ]$	$G'_{T1}(s) = K_0 \frac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s+1)(T_2 s+1)}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_2' = 0$, $B_2' = B_2'$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_2{}' \pm\right.$ $\pm\sqrt{(0{,}5+B_2{}')^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $\varepsilon = 0{,}866$
3	$m_x K_x << 1$, $K_{bz}n_z K_z << 1$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_3 = B_3$	$T_{4,5} = 0{,}5\tau[0{,}5 + A_2 \pm$ $\pm\sqrt{(0{,}5+A_2)^2 - 1/3}\ ]$ $T_{1,2} = 0{,}5\tau[0{,}5 + B_3 \pm$ $\pm\sqrt{(0{,}5+B_3)^2 - 1/3}\ ]$		$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{yy}n_y}$ $A_2' = 0$, $B_3' = B_3'$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_3{}' \pm\right.$ $\pm\sqrt{(0{,}5+B_3{}')^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $\varepsilon = 0{,}866$
3	$K_{xy} << 1$	$K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_4 = B_4$	$T_{4,5} = 0{,}5\tau[0{,}5 + A_2 \pm$ $\pm\sqrt{(0{,}5+A_2)^2 - 1/3}\ ]$ $T_{1,2} = 0{,}5\tau[0{,}5 + B_4 \pm$ $\pm\sqrt{(0{,}5+B_4)^2 - 1/3}\ ]$		$K_0 = \frac{K_{F_{x1}}}{1+K_{yy}n_y}$ $A_2' = 0$, $B_4' = B_4'$	$T_{1,2} = 0{,}5\tau\left[0{,}5 + B_4{}' \pm\right.$ $\pm\sqrt{(0{,}5+B_4{}')^2 - 1/3}\ ]$ $T_3 = 0{,}289\tau$ $\varepsilon = 0{,}866$
4	**Using the first element of Padé Approximation for** $e^{-s\tau}$**:** $G_{T3}(s) = K_0 \frac{T_2 s+1}{T_3 s+1}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1 = m_x K_x + m_y K_y K_{\kappa_r}$, $B_1 = B_1$	$T_0 = \tau$, $T_1 = 0{,}5\tau$ $T_2 = \tau(0{,}5 + A_1)$ $T_3 = \tau(0{,}5 + B_1)$	**Using the first element of Padé Approximation for** $e^{-s\tau}$**:** $G_{T3}(s) = K_0 \frac{T_2 s+1}{T_3 s+1}$	$K_0 = \frac{K_{F_{x1}}}{1+K_{xy}n_x + K_{bz}n_z K_z + K_{yy}n_y}$ $A_1' = m_x K_x$, $B_1' = B_1'$	$T_0 = \tau$, $T_1 = 0{,}5\tau$ $T_2 = \tau(0{,}5 + A_1')$ $T_3 = \tau(0{,}5 + B_1')$

Table 1. Operator transmittances, coefficients of gain and time constants of generalized and simplified MM of dynamic system of turning of shafts with low rigidity in elastic-deformable condition (continued)

1	2	3	4	5	6	7
4	$m_x K_x << 1$	$K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_2 = B_2$	$T_0 = \tau$, $T_1 = 0{,}5\tau$ $T_2 = \tau(0{,}5 + A_2)$ $T_3 = \tau(0{,}5 + B_2)$		$K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{bz} n_z K_z + K_{yy} n_y}$ $A_1' = 0$, $B_2' = B_2'$	$T_0 = \tau$ $T_1 = T_2 = 0{,}5\tau$ $T_3 = \tau(0{,}5 + B_2')$
	$m_x K_x << 1$, $K_{bz} n_z K_z << 1$	$K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{yy} n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_3 = B_3$	$T_0 = \tau$, $T_1 = 0{,}5\tau$ $T_2 = \tau(0{,}5 + A_2)$ $T_3 = \tau(0{,}5 + B_3)$		$K_0 = \frac{K_{F_{x1}}}{1 + K_{xy} n_x + K_{yy} n_y}$ $A_2' = 0$, $B_3' = B_3'$	$T_0 = \tau$ $T_1 = T_2 = 0{,}5\tau$ $T_3 = \tau(0{,}5 + B_3')$
	$K_{xy} << 1$	$K_0 = \frac{K_{F_{x1}}}{1 + K_{yy} n_y}$ $A_2 = m_y K_y K_{\kappa_r}$, $B_4 = B_4$	$T_0 = \tau$, $T_1 = 0{,}5\tau$ $T_2 = \tau(0{,}5 + A_2)$ $T_3 = \tau(0{,}5 + B_4)$		$K_0 = \frac{K_{F_{x1}}}{1 + K_{yy} n_y}$ $A_2' = 0$, $B_4' = B_4'$	$T_0 = \tau$ $T_1 = T_2 = 0{,}5\tau$ $T_3 = \tau(0{,}5 + B_4')$

As follows from the structural schematic of *DS* (Fig. 1), in the control object there are closed circuits determined by the specifics of grinding of parts „following the feed ridge” and by the effect of elastic deformation of the *DS* along axes X and Y. The system of equations [5] permits determination of transmittance *DS* - *CO* for any of the input variables, both with respect to the control effects and to the interference effects.

On the example of one input variable in the form of elastic deformation of the system in the radial direction $g_y(s)$ and of an input effect – tensile force $F_{x1}(s)$, the structural schematic is transformed to the form shown in Fig. 2, and the transmittance is defined by the following expression:

$$G_{sc}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{1 + A_1(1-e^{-s\tau})}{1 + B_1(1-e^{-s\tau})}, \tag{18}$$

where:

$$K_0 = K_{F_{x1}} \cdot \frac{1}{1 + K_{xy} \cdot n_x + K_{yy} \cdot n_y}, \tag{19}$$

$$A_1 = m_x \cdot K_x, \tag{20}$$

$$B_1 = \frac{m_x \cdot K_x}{1 + K_{xy} \cdot n_x + K_{yy} \cdot n_y}. \tag{21}$$

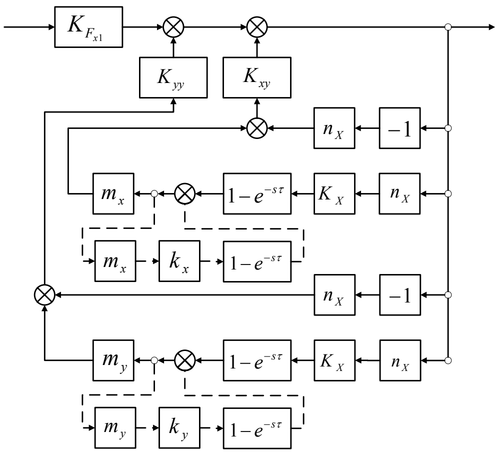

Fig.2. Transformed structural schematic of the object with respect to control effect F_{x1}

Comparing the obtained relations with transmittances of DS of turning of low-rigidity shafts in the elastic-deformable condition and taking into account that for the process of grinding the coefficients $K_{\kappa_r} = 0$ and $K_{bz} = 0$, one can observe that the presented *MM* can be considered as a special case of the mathematical model of *DS* of turning (with the grinding wheel considered as a cutting edge with $\kappa_r = 90^\circ$).

For known numerical values of coefficient of gain $m_x, m_y, K_{xy}, K_{yy}, n_x, n_y$ and of lag time τ, calculated analytically based on prior information or determined experimentally, the relations for transmittance parameters can be significantly reduced if it is permitted to leave out the feedback in the object marked with broken line in Fig 2 ($m_x K_x << 1$), then $A_1 = 0$, $B_1 = 1/(1 + K_{xy} \cdot n_x + K_{yy} \cdot n_y)$.

Splitting the exponential function $e^{-s\tau}$ into a Padé series permits equivalent presentation of the *MM* (18) by means of transmittances of typical dynamic components. Using the first two elements of the Padé series we can write [4, 5]:

$$G_{sc}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{T_3^2 \cdot s^2 + T_3' \cdot s + 1}{(T_1 s + 1) \cdot (T_2 s + 1)}, \qquad (22)$$

where:

$$T_{1,2} = 0{,}5\tau \cdot \left[0{,}5 + B_2 \pm \sqrt{(0{,}5 + B_2)^2 - 1/3} \right], \qquad (23)$$

$$T_3 = 0{,}289\tau, T_3' = (0{,}5 + A_1) \cdot \tau. \qquad (24)$$

Further transformation of the numerator of transmittance in accordance with expression (22) is performed in a manner analogous to cases of *DS* of turning as above.

In particular, for $A_1 = 0$ the transmittance numerator gets transformed to the form:

$$T_3^2 \cdot s^2 + 2\varepsilon T_3 \cdot s + 1,$$

where: $T_3 = 0{,}289\tau, \varepsilon = 0{,}866$.

Depending on the value of coefficient $A_1 < 0{,}077$, transmittance can be written in the following typical form:

$$G_{sc}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{T_3^2 \cdot s^2 + 2\varepsilon T_3 \cdot s + 1}{(T_1 s + 1) \cdot (T_2 s + 1)}, \qquad (25)$$

where: $\varepsilon = (0{,}5 + A)/0{,}577$ - coefficient of attenuation.

In the case when $A_1 \geq 0{,}078$, the approximating relation for the analysed transmittance assumes the form:

$$G_{F_{x1}}(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{(T_4 \cdot s + 1) \cdot (T_5 \cdot s + 1)}{(T_1 \cdot s + 1) \cdot (T_2 \cdot s + 1)}, \tag{26}$$

where:

$$T_{4,5} = 0{,}5\tau \cdot \left[0{,}5 + A_1 \pm \sqrt{(0{,}5 + A_1)^2 - 1/3} \right]. \tag{27}$$

By analogy, based on the generalized structural schematic fragmentary model of the *DS* were obtained for elastic regulatory effects. The approcimating relations of transmittance of the *DS* of oscillation grinding, in this case for various control effects differ from the quoted expressions only in the value of the coefficient of gain K_0 (*CO*, relations for calculations to be found in reference [13]). Operator transmittances, coefficients of gain and time constants for the generalized and reduced *MM* of the dynamic system of straight grinding of shafts in the elastic-deformable state, taking into account the use of one or two segments of splitting of function $e^{-s\tau}$ into a Padé series, are given in Table 2.

4. IDENTFICATION OF DS OF PLUNGE GRINDING OF SHAFTS WITH LOW RIGIDITY

As the input effects on the object one of the effects mentioned earlier is adopted, that generate the elastic-deformable state - $K_{F_{x1}}, K_e, e, K_{Mi}$ and the rate of travel of the cross slide v_{pop}, and as the output effects – elastic deformations of the technological system with relation to coordinate Y.

The mutual connections between the grinding forces and the thickness of the machined layer with the surface of the machined part, as in the models analysed above, are considered to be non-inertial [9]. To the initial assumptions and conditions adopted earlier we should add:

2 the grinding is performed at constant machining speed, at invariable grinding parameters of the grinding wheel and properties of the material of the machined part;

2 the grinding width $b = const$ and it is equal, in plunge grinding, to the width of the machined part or of the grinding wheel.

Apart from this, considered are only linear deformations of the system and variability of rigidity K_y and K_z with relation to axes Y and Z. With the adopted assumptions, the force of machining is determined only by the thickness of the machined layer $a(t)$:

$$F_\zeta = m_\zeta \cdot a(t), \tag{28}$$

where: $\zeta \in \{Y, Z\}$.

MM of the technological system of plunge grinding of elastic-deformable shafts of low rigidity can be presented as the system of equations:

In this system of equations it was taken into account that elastic deformation along axis Z lead to changes in the thickness of machined layer and may be considered as additional components of increment g_y. The expression for coefficient K_{bz}, determining the bonds between increment of machining depth b and force F_c, were obtained earlier - $K_{bz} = \sin(g_{z0} / R \approx g_{z0} / R$ [4].

$$\left.\begin{aligned}
&F_{\zeta}(s) = m_{\zeta} \cdot a(s),\\
&g_y(s) = K_{yy} \cdot F_p(s) + K_{F_{x1}} \cdot F_{x1}(s) + K_e \cdot e(s) + K_{Mi} \cdot M_i(s) + K_{bz} \cdot g_z(s),\\
&g_z(s) = K_z \cdot F_c(s),\\
&a(s) = \frac{1}{s}(1 - e^{-s\tau}) \cdot v_{pop}(s) - (1 - e^{-s\tau}) \cdot g_y(s).
\end{aligned}\right\} \tag{29}$$

In accordance with the system of equations (29) a generalized structural schematic was built for cylindrical plunge grinding of elastic-deformable shafts (Fig. 3a). The structural schematic of transformation to the input parameter $g_y(s)$ is presented in Fig. 3b. In this case, the transmittance of *DS* as a control object is written as:

$$G_{sc}{}'(s) = \frac{g_y(s)}{F_{x1}(s)} = K_0 \cdot \frac{1}{1 + B_9 \cdot (1 - e^{-s\tau})}, \tag{30}$$

where: $K_0 = K_{F_{x1}}, B_9 = B_2 = K_{yy} \cdot m_y + K_{bz} \cdot K_z \cdot m_z$, if we do not include the effect of increment of component F_c of machining force on elastic deformations along coordinate Y ($m_z \cdot K_{bz} \cdot K_z << 1$), then $B_{10} = K_{yy} \cdot m_y$.

After the transformations we obtain:

$$G_{sc}{}'(s) = K_0 \cdot \frac{T_3^2 \cdot s^2 + 2\varepsilon T_3 \cdot s + 1}{(T_1 \cdot s + 1) \cdot T_2 \cdot s + 1)}, \tag{31}$$

where: $T_3 = 0{,}289\tau, \varepsilon = 0{,}866,$

$$T_{1,2} = 0{,}5\tau \cdot \left[0{,}5 + B_2 \pm \sqrt{(0{,}5 + B_2)^2 - 1/3}\right].$$

For input effects M_i and e of transmittance, *CO* are also determined in accordance with (31), but the coefficient of gain K_0 is determined on the basis of relation [13].

In a number of cases, with an accuracy that is sufficient for engineering calculations, it is advisable to retain in the approximating relations for the transmittance (30) the first segment of splitting of the function $e^{-s\tau}$ into a Padé series [5, 13], and then the transmittance (30), after the transformations, is reduced to the form of typical dynamic elements, as in the case of turning.

The operator transmittances, coefficients of gain and time constants for the generalized and simplified MM of the dynamic system of plunge grinding of elastic-deformable shafts, taking into account the use of one and two segments of splitting the functions $e^{-s\tau}$ into a Padé series are presented in Table 2.

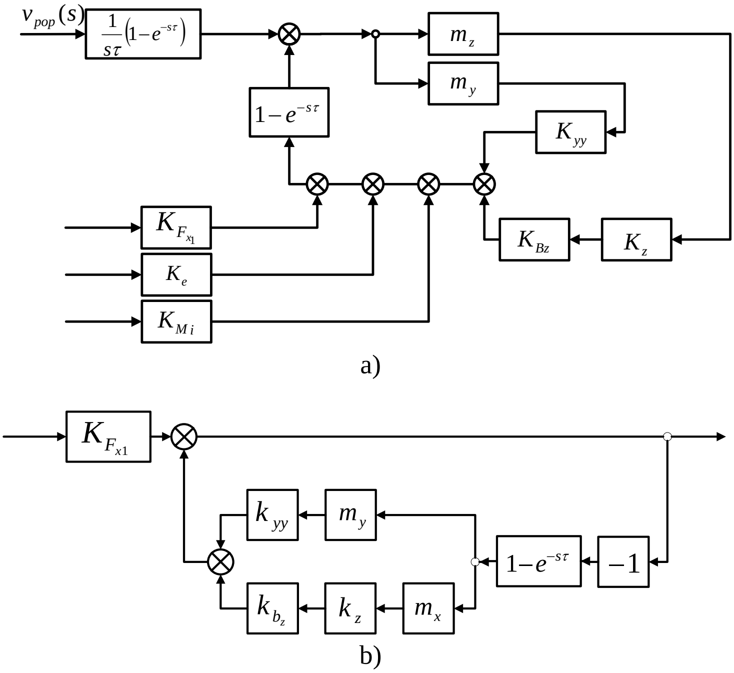

Fig.3. Structural schematics of *DS* in plunge grinding of elastic-deformable shafts with low rigidity: a) generalized, b) structural

5. HIERARCHICAL LEVELS AND TYPICAL STRUCTURES OF DS OF PROFILING LOW-RIGIDITY SHAFTS IN ELASTIC-DEFORMABLE STATE

The systemic categories include objects and processes composed of individual parts, elements and objects characterized by the ability of complete (full) functioning, with any technological system being distinguished by a certain set of characteristics:

1. connections of the system with the surroundings (in the considered case those are connections with interference and with the initial error of semi-finished product);
2. functions realized by the system, i.e. assurance of machining precision parameters defined in the technological process;
3. structure of the system;

4. all of the functional properties and the system, defined as the character of the transmittance.

The listed set of characteristics is fully relevant to technological systems of elastic-deformable shafts with low rigidity and their *MM* taking into account the properties of control system under stabilised and dynamic conditions. When considering *MM* of dynamic systems of machining of elastic-deformable parts as systemic objects taking into account the principally important and at the same time different specific features of control object functioning, the information-oriented approach is advisable. Such an approach shows that the generalized mathematic model of *DS* of straight turning is the most complete, has a high level of information content in the structure hierarchy and MM of dynamic systems, and is situated at the 1st – highest hierarchy level (Fig. 4).

The generalized mathematical model $G_T(s)$, situated on the 1st level of hierarchy, corresponds to the system of equations and the structural schematic [13].

On the 2nd level of hierarchy the generalized *MM,* with relation to the degree of accuracy of approximation of function $e^{-s\tau}$ in its splitting into a Padé series, can be divided into two models, the first of which corresponds to relation (6) and includes the first two segments of splitting of the function $e^{-s\tau}$.

On the 3rd level of hierarchy the *MM* of dynamic system of straight turning of elastic-deformable shaft with low rigidity is divided into two models, for the first of which coefficient $K_{\kappa_r} \neq 0$, $\kappa_r \neq 90^\circ$, and the second $K_{\kappa_r} = 0$, $\kappa_r = 90^\circ$, and transmittances are $G_T\ (s), G_{T3}\ (s)$ and coefficients A_1, A_1' and B_1, B_1' , respectively, into which K_{κ_r} is introduced, (Tables 1 and 2). At this level there are also the *MM* if dynamic system of processes of oscillation grinding $G_{sc}(s)$ and external plunge grinding $G_{sc}'(s)$ that can be considered as special cases of *MM* of *UD* of turning, taking into account that for oscillation grinding $K_{\kappa_r} = 0$, $K_{bz} = 0$, and $K_{\kappa_r} = 0$, $m_x K_x = 0$, $K_{xy} = 0$ respectively for external plunge grinding (Tables 1 and 2).

On the 4th level of hierarchy there are *MM* of dynamic system of turning $G_T(s), G_{T1}'(s), G_{T2}(s), G_{T2}'(s), G_{T3}(s)$, differing in the values of coefficients A_1, A_2, A_1' i A_2' , for *DS* of oscillation grinding $G_{sc}(s)$ and external plunge grinding $G_{sc}'(s)$ (Tables 1 and 2).

On the 5th level of hierarchy there are *MM* of dynamic system obtained without taking into account the effect of the link - shown in the Figure with broken line - on the dynamics; this is the effect of closed contour feedback by coefficient $m_x K_x << 1$ and transmittance $(1 - e^{-s\tau})$ on the increment of machined layer thickness.

On level 6 of hierarchy there are fragmentary *MM* of dynamic system without inclusion of one contour of internal feedback of $m_x K_x << 1$ and without taking into account the effect of elastic deformation along axis Z on the depth of machining $K_{bz} K_z n_z << 1$ (Tables 1 and 2).

On the 7th – lowest – level of hierarchy there are fragmentary *MM* of dynamic system of machining, taking into account only the effect of elastic deformations along axis Y on the increment of machined layer thickness $(K_{xy} << 1)$ in turning (Tab. 1), in external plunge grinding, and two different fragmentary models for oscillation grinding (Tab. 2).

Fig. 5 presents typical structures of *DS* of profiling elastic-deformable shafts of low rigidity for a variety of control force effects, for which typical transmittances of *CO* are given in Tables 1 and 2, and coefficients of gain relative to the effects are determined in accordance with relations given in [13]. For *DS* of straight turning with the inclusion of suitable regulatory effects all of the presented structures can be applied, for oscillation grinding – structures as in Fig. 3.17 a, b, h and i, and for external plunge grinding – structures as per Fig. 3.17 h, i.

Table 2. Operator transmittances, coefficients of gain and time constants of *DS MM* in grinding of shafts in elastic-deformable state

	Operator transmittance of DS MM in grinding of shafts in elastic-deformable state	**Coefficients of Gain**	**Time Constants**
1	**2**	**3**	**4**
1	**Using first two elements of Padé Approximation for $e^{-s\tau}$:** $G_{sc}(s)=K_0\dfrac{T_3^2s^2+T_3^{'}s+1}{(T_1s+1)(T_2s+1)}$	$K_0=\dfrac{K_{F_{x1}}}{(1+K_{xy}n_x+K_{yy}n_y}$ $A_1=m_xK_x$ $B_1=\dfrac{m_xK_x}{(1+K_{xy}n_x+K_{yy}n_y)}$	$T_3=0{,}289\tau$ $T_3^{'}=(0{,}5+A_1)\tau$ $T_{1,2}=0{,}5\tau\big[0{,}5+B_1\pm$ $\pm\sqrt{(0{,}5+B_1)^2-1/3}\ \big]$
1	$m_xK_x<<1$ $G_{sc}(s)=K_0\dfrac{T_3^2s^2+2\varepsilon T_3s+1}{(T_1s+1)(T_2s+1)}$	$K_0=\dfrac{K_{F_{x1}}}{(1+K_{xy}n_x+K_{yy}n_y}$ $A_1=0$ $B_2=\dfrac{K_xn_x(m_xK_{xy}+m_yK_y)}{(1+K_{xy}n_x+K_{yy}n_y)}$	$T_3=0{,}289\tau$ $\varepsilon=0{,}866$ $T_{1,2}=0{,}5\tau\big[0{,}5+B_2\pm$ $\pm\sqrt{(0{,}5+B_2)^2-1/3}\ \big]$
2	$K_{xy}<<1$ $G_{sc}(s)=K_0\dfrac{T_3^2s^2+T_3^{'}s+1}{(T_1s+1)(T_2s+1)}$	$K_0=\dfrac{K_{F_{x1}}}{(1+K_{yy}n_y)}$ $A_1=m_xK_x$ $B_3=m_xK_x/(1+K_{yy}n_y)$	$T_3=0{,}289\tau$ $T_3^{'}=(0{,}5+A_1)\tau$ $T_{1,2}=0{,}5\tau\big[0{,}5+B_3\pm$ $\pm\sqrt{(0{,}5+B_3)^2-1/3}\ \big]$
2	$m_xK_x<<1, K_{xy}<<1$ $A_1<0{,}077$	$K_0=\dfrac{K_{F_{x1}}}{(1+K_{yy}n_y)}$ $A_1=0$ $B_4=\dfrac{m_xK_x\cdot m_yK_{yy}}{(1+K_{yy}n_y)}$	$T_3=0{,}289\tau$ $\varepsilon=0{,}866$ $T_{1,2}=0{,}5\tau\big[0{,}5+B_4\pm$ $\pm\sqrt{(0{,}5+B_4)^2-1/3}\ \big]$ $\varepsilon_1=(0{,}5+A_1)/0{,}577$
3	$A_1\geq 0{,}078$ $G_{sc}(s)=K_0\dfrac{(T_4s+1)(T_5s+1)}{(T_1s+1)(T_2s+1)}$	$K_0=\dfrac{K_{F_{x1}}}{(1+K_{xy}n_x+K_{yy}n_y}$ $A_1=m_xK_x$ $B_5=B_3=\dfrac{m_xK_x}{(1+K_{yy}n_y)}$	$T_{4,5}=0{,}5\tau\big[0{,}5+A_1\pm$ $\pm\sqrt{(0{,}5+A_1)^2-1/3}\ \big]$ $T_{1,2}=0{,}5\tau\big[0{,}5+B_5\pm$ $\pm\sqrt{(0{,}5+B_5)^2-1/3}\ \big]$
4	**Using the first element of Padé Approximation for $e^{-s\tau}$:** $G_{sc1}(s)=K_0\dfrac{(T_{02}s+1)}{(T_{03}s+1)}$	$K_0=\dfrac{K_{F_{x1}}}{(1+K_{xy}n_x+K_{yy}n_y}$ $A_1=m_xK_x$ $B_6=B_3=\dfrac{m_xK_x}{(1+K_{yy}n_y)}$	$T_0=\tau$ $T_{01}=0{,}5\tau$ $T_{02}=(0{,}5+A_1)\tau$ $T_{03}=(0{,}5+B_6)\tau$
4		$K_0=\dfrac{K_{F_{x1}}}{(1+K_{yy}n_y)}$ $A_1=0$	$T_0=\tau$ $T_{01}=T_{02}=0{,}5\tau$

Table 2. Operator transmittances, coefficients of gain and time constants of *DS MM* in grindingof shafts in elastic-deformable state (cont.)

1	2	3	4
4		$B_7 = B_2 = \dfrac{K_x n_x (m_x K_{xy} + m_y K_y)}{(1 + K_{xy} n_x + K_{yy} n_y)}$ $B_8 = B_4 = \dfrac{m_x K_x \cdot m_y K_{yy}}{(1 + K_{yy} n_y)}$	$T_{03} = (0{,}5 + B_7)\tau$ $T_{02} = (0{,}5 + B_8)\tau$
5	**Plunge grinding** **Using first two elements of Padé Approximation for** $e^{-s\tau}$**:** $G'_{sc}(s) = K_0 \dfrac{T_3^2 s^2 + 2\varepsilon T_3 s + 1}{(T_1 s + 1)(T_2 s + 1)}$	$K_0 = K_{F_{x1}}$ $B_9 = K_{yy} m_y + K_{bz} K_z m_z$ $B_{10} = K_{yy} m_y$	$T_3 = 0{,}289\tau$ $\varepsilon = 0{,}866$ $T_{1,2} = 0{,}5\tau\big[0{,}5 + B_9 \pm$ $\pm\sqrt{(0{,}5 + B_9)^2 - 1/3}\ \big]$
6	**Using the first element of Padé Approximation for** $e^{-s\tau}$**:** $G'_{sc1}(s) = K_0 \dfrac{(T_{01} s + 1)}{(T_{03} s + 1)}$	$K_0 = K_{F_{x1}}$ $B_9 = K_{yy} m_y + K_{bz} K_z m_z$ $B_{10} = K_{yy} m_y$	$T_0 = \tau, T_{01} = 0{,}5\tau$ $T_{03} = (0{,}5 + B_9)\tau$ $T_{03} = (0{,}5 + B_{10})\tau$

CONCLUSION

As follows from the performed study, dynamic structures of MM of technological systems for low-rigidity shafts with control of their elastic-deformable condition include, apart from inertial segments characteristic for MM of feed-related control, also overload segments. The occurrence of the overload segments in transmittances of the MM reduces the inertness of the control objects with respect to channels of control of additional force effects. według kanałów sterowania dodatkowymi oddziaływaniami siłowymi. For example, with close values of time constants of the numerator and denominator in relations (16) and (26), as happens is numerous cases, the properties of model of CO approach those of the non-inertial segment with transmission coefficient K_0.

It should be emphasized that the discussed mathematical description of the *CO* was made with the exclusion of "small" time constants characterizing the dynamic properties of the process of machining and of the equivalent elastic system. Such an approach is justified as the *ACS* or *AC* circuit includes, apart from the object, also an automatic control device and other components with "large" time constants, whose dynamic properties are highly significant in the solution of the problem of stability analysis and synthesis of corrective segments.

Comparison of MM of the object for various control effects permits the statement that with the application of additional force effects the object has a notably lower inertness compared to the case of control focused on the feed channel. Thanks to this in the *ACS* and *AC* of the elastic-deformable state of parts higher indexes of control quality can be achieved in the dynamics and there is a possibility of effective counteraction of interference caused by changes in material allowance for machining and in the hardness of machined semi-finished products by varying their rigidity on the length of machining.

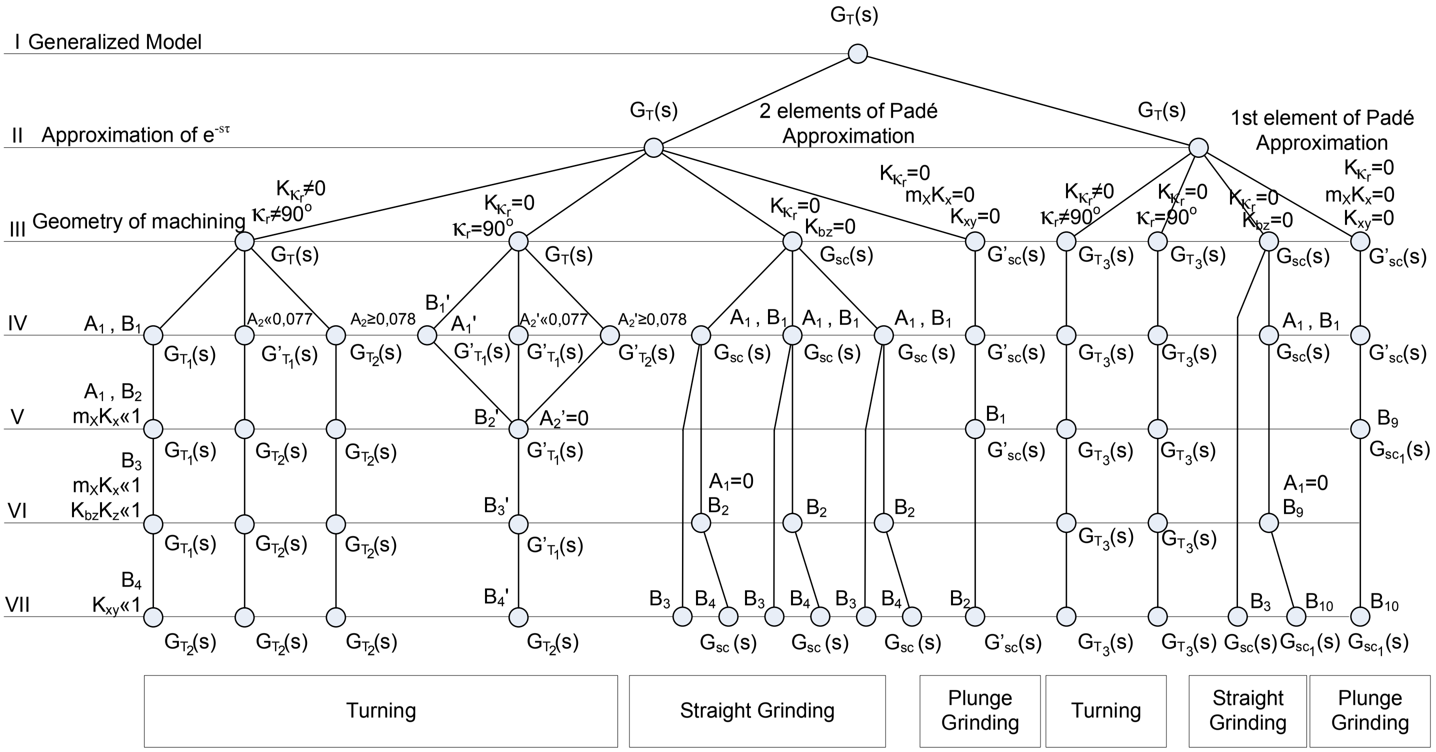

Fig.4. Hierarchical levels of MM of dynamic system of machining of elastic-deformable shafts of low rigidity

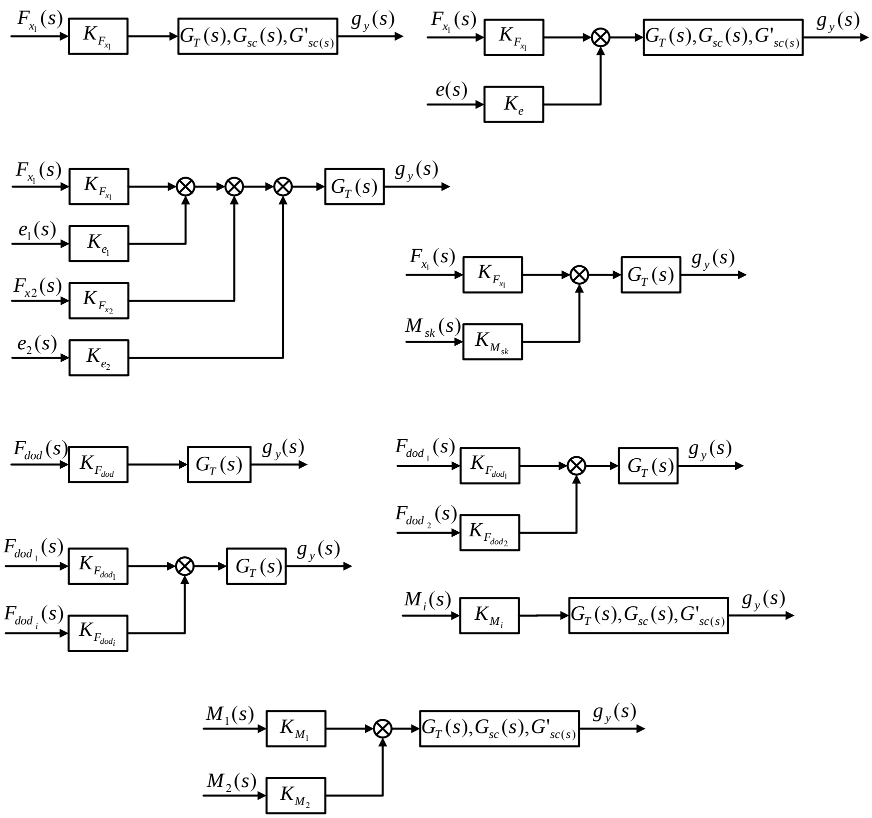

Fig.5. Typical structures of MM of dynamic system of machining of elastic-deformable shafts with low rigidity

The presented analysis shows that the information-oriented method is also suitable as it permits division of MM of dynamic systems – control objects according to the quality and quantity of information introduced in the MM and creates the possibility of systemizing the capacity of design engineers in the design and technological development of technological processes, development of systems of automated design, as well as of ACS and AC, with a view to solving the problems they are facing.

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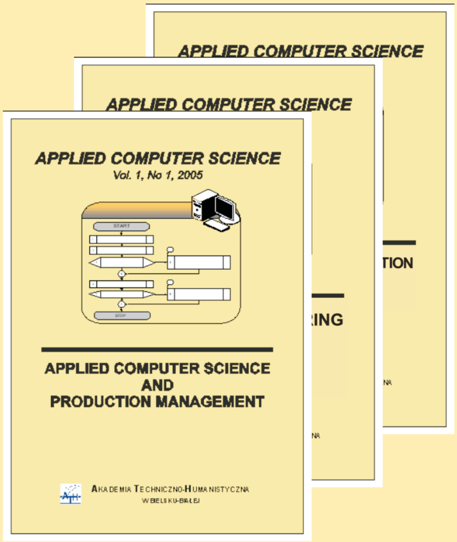

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